Application of an Evolution Strategy in Celestial Mechanics

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Introduction

In general, evolutionary algorithms comprise the two branches genetic algorithms [1] and evolution strategies (ES) [4], both of which were invented in the 1960s and 1970s. Here we focus solely on ES which have seen many improvements within the last decades and can now be regarded as a real alternative to standard optimization techniques in many areas, especially in cases where gradient methods like classical least-squares algorithms fail.

Compared to other optimization techniques, ES are easy to adapt to diverse problems, because one rarely needs any a priori insight into the mathematical/physical nature of the optimization task. Once implemented, the same algorithm can be applied to a wide range of problems without substantial changes. The only necessary condition for ES to successfully operate on a given specific problem is the inherent existence of strong causality, which here means that similar causes lead to similar results, i.e., there is no (short-term) chaotic behavior in the underlying system.

In the following sections we present examples for the application of an ES with covariance matrix adaptation (ES-CMA) [2]. All strategy parameters were chosen empirically here; but this could be avoided by the implementation of a Meta-ES, that eventually will, in addition to the problem specific unknowns, optimize its own strategy parameters automatically.

Application #1: gravity field determination

The goal is to find spherical harmonic coefficients c_{nm} and s_{nm} up to a given maximum degree n_{max} and order m_{max}, representing an nxm gravity field of a central (celestial) body, e.g. planet Earth. As an example, here we solve for a 4x4 gravity field, which is equivalent to a 21-d optimization problem.

Earth's gravity field directly influences the motion of an orbiting satellite (test mass). To determine the coefficients, a set of N (simulated/measured) satellite positions $\mathbf{r}_i^s$ is given. We search for an optimal set of spherical harmonics, leading to calculated positions $\mathbf{r}_i^c$. Comparing them with the simulated ones yields deviations $\Delta\mathbf{r}_i = \mathbf{r}_i^s - \mathbf{r}_i^c$, which shall not exceed a chosen threshold value.

Depending on the norm, the performance index Q (objective function, or quality) may be defined as $Q = \sum_{i=1}^{N} \|\Delta\mathbf{r}_i\| \to \min$. The termination quality was set to $Q^* = 1/1000\, mm$, and for $N = 90$ a (1,40)-ES-CMA was realized. The values in the round bracket indicate that in each new generation there is only 1 parent creating an offspring of 40 individuals, and only the (mutated) offspring is subject to selection afterwards. Fig. 1 illustrates the evolution of the unknowns. Adaptation phases to escape from local optima are clearly visible.

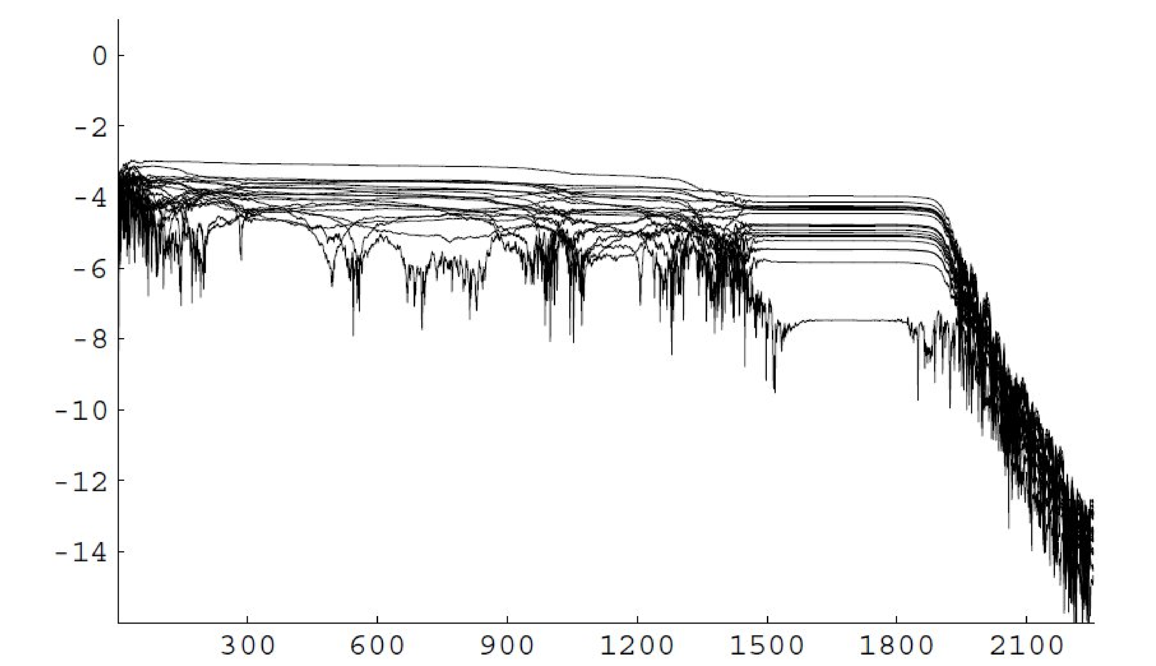

Fig. 1: Logarithm of the absolute residual values of the unknowns versus generation number.

The following table provides final/optimized values (all in units of 10^{-10}). Any digits identical with the original harmonics (as used within our simulation) are in bold print.

n	m	c_{nm}	s_{nm}
2	0	−4841695.4834480	-
3	0	+9571.7060002975	-
4	0	+5397.7705833457	-
2	1	−1.8694714700433	+11.954500954474
3	1	+20301.372076698	+2481.3079540691
4	1	−5362.4358305647	−4737.7249759825
2	2	+24392.609849473	−14002.665205972
3	2	+9047.0636114776	−6189.2285463862
4	2	+3506.7012168619	+6625.7136424735
3	3	+7211.4491711647	+14142.039502771
4	3	+9908.6882512345	−2009.8746087090
4	4	−1884.8146556533	+3088.4815006772

Application #2: satellite orbit from two positions

We want to find the solution to a seemingly simple boundary value problem. Given are two position vectors $\mathbf{r}_A, \mathbf{r}_B$ valid at epochs t_A, t_B (with $t_B > t_A$, to fix the sense of direction for a satellite's motion), and a known force field, e.g., a 8x8 gravity field of the primary body, cf. fig 2. Thus, we face a non-Keplerian motion problem (orbit determination).

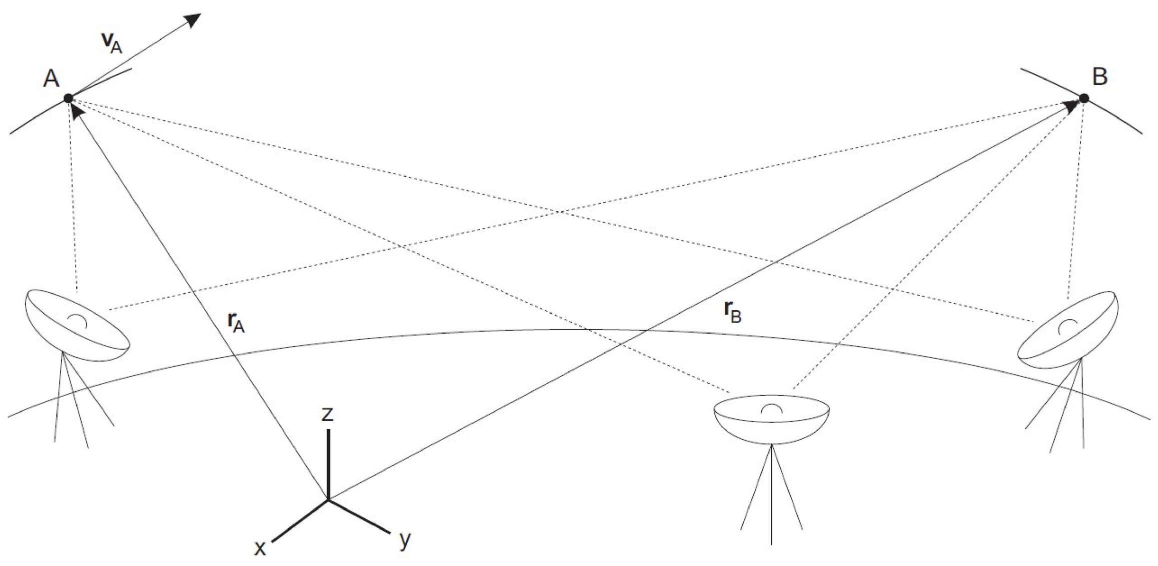

Fig. 2: Boundary value problem: given satellite positions, e.g., via observations from ground sites.

Now, the task is to transform the original boundary value problem into an initial value problem, i.e., to solve for the correponding initial velocity vector $\mathbf{v}_A$.

Then, knowing the initial state and arc length (time of flight $t_B - t_A$), the orbit between A and B can be determined via usual integration methods. There exist only 3 unknowns: $\mathbf{v}_A$'s cartesian components. Solving this problem via ES does not make use of any further theoretical knowledge on celestial mechanics (availability of integrals of motion etc.).

We simply have to define a suitable performance index, e.g, $Q = \|\Delta\mathbf{r}_B\| = \|\mathbf{r}_B - \mathbf{r}_B^{NI}\| \to \min$, NI denotes classical numerical integration. Again, a (1,40)-ES-CMA was employed for optimization, with termination quality set to $Q^* = 1 \cdot 10^{-10} mm$. Fig. 3 depicts the residuals and quality for a given numerical example, the final solution of which was found after only 145 generations. It also illustrates the start of the optimization via some intermediate resulting orbits (generations are gray level coded).

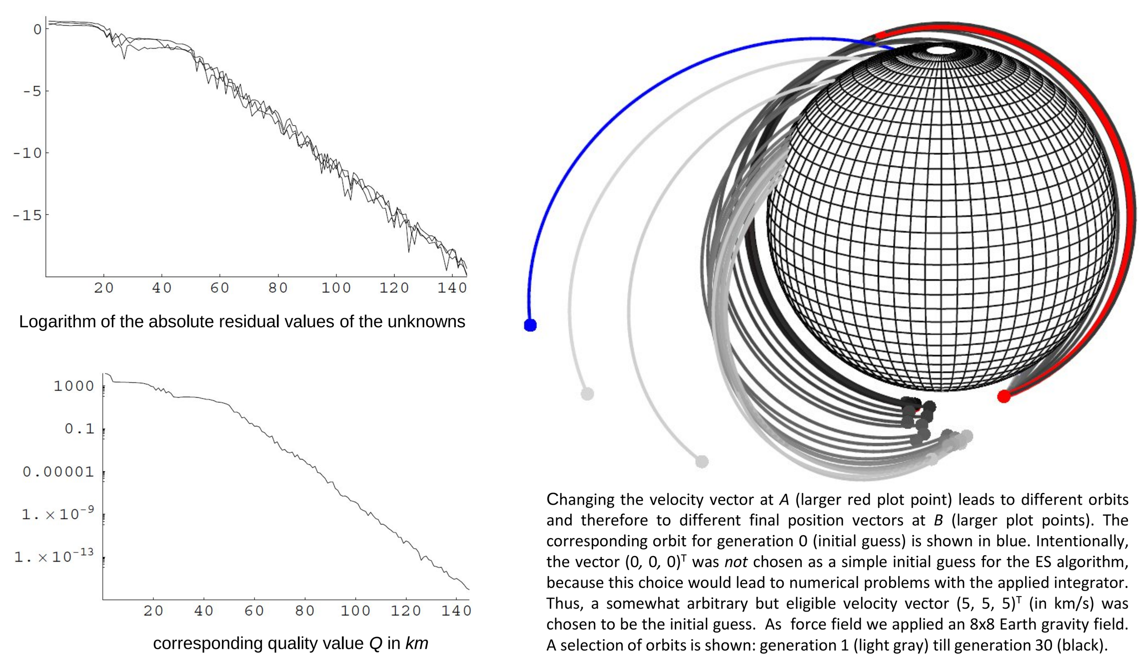

Fig. 3: Residuals (top left) and quality (down left) vs. generation #, as well as resulting obits (right).

Application #3: spectral analysis w/ trend & offset

In a more general application, we demonstrate the simultaneous (!) determination of both polynomial and periodical time series parameters, all of which are allowed to be real-valued (beneficial for an efficient representation). Given is a tabulated time series (t_k, y_k) via k_{max} (not necessarily equidistant) data points.

In our simple example, we fixed a certain set of parameters and simulated the data for a number of $k_{max} = 100$ randomly chosen epochs, cf. fig 4.

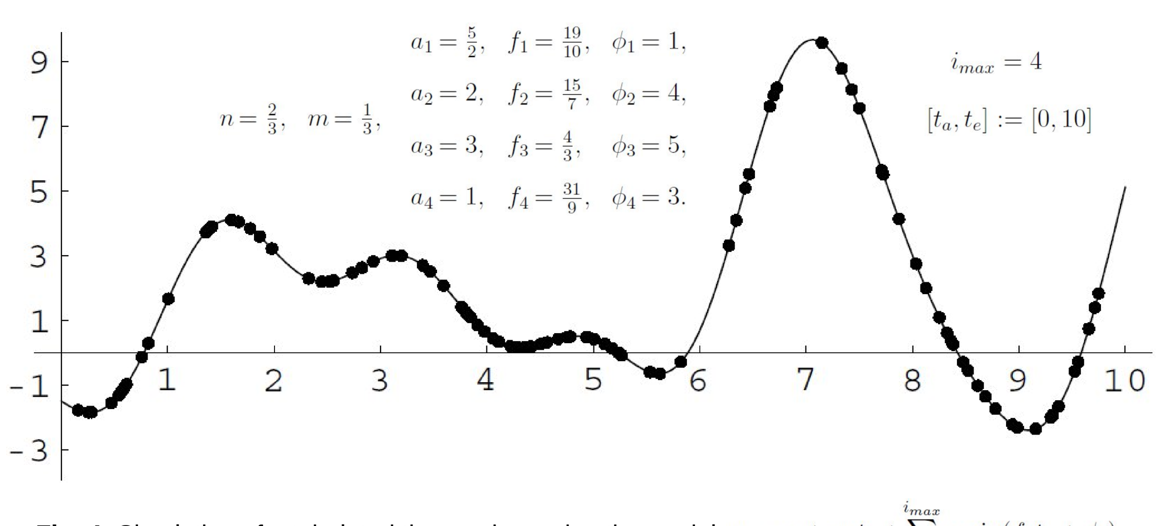

Fig. 4: Simulation of a tabulated time series, using the model $y_k = n + m\,t_k + \sum_{i=1}^{i_{max}} a_i \sin(f_i t_k + \phi_i)$.

This time we chose a classical performance index, namely the least-squares approach, i.e., minimize the sum of the squared residuals $v_k := y_k - y_k^{ES}$.

The working precision was set to 16 significant digits. The termination quality shall be equivalent to the precision of the data, therefore $Q^* = 10^{-16}$.

To solve our 14-dimensional optimization problem, we used a (1,10)-ES-CMA. All initial amplitudes and frequencies were set equal to 1, and all of the remaining initial parameter values were simply set to zero. Thus, our initial guess represents a single harmonic oscillation (sine-wave w/ amplitude = 4). It took 7945 generations to find a solution, cf. fig 5.

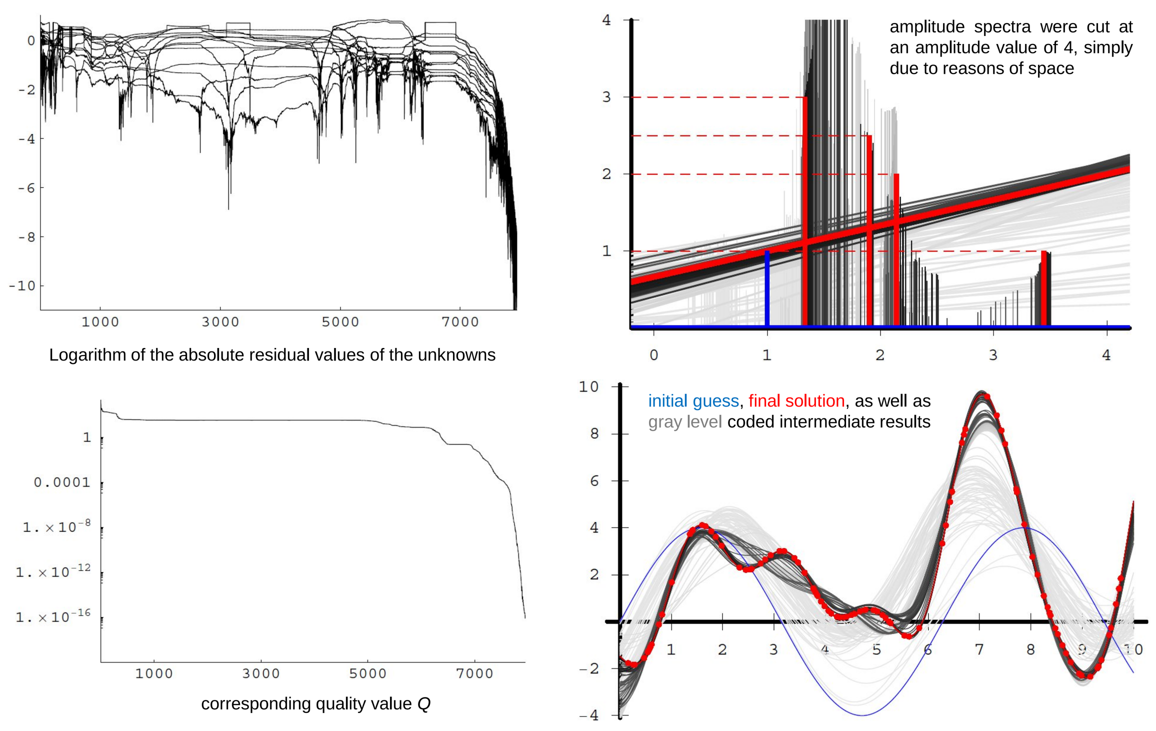

Fig. 5: Residuals (top left) and quality (down left) vs. generation #, as well as the resulting amplitude spectra superimposed by a depiction of trend and offset (top right), and resulting signals (down right).

Outlook

Selected applications of relatively simple ES were presented. In future, this stochastic optimization technique should come to further attention within the field of celestial mechanics, especially for the direct solution of inverse problems. Ever growing hardware and software capabilities will support this approach. For example, the author plans to use ES for improved asteroid modeling within the construction of a new solar-system ephemeris [3].

References

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Acknowledgements

This research was funded by German Research Foundation (DFG), within research unit FOR 1503 "Space-Time Reference Systems for Monitoring Global Change and for Precise Navigation in Space".

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7 – 12 September 2014

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